

Muonium-Antimuonium Conversion in Models with Dilepton Gauge Bosons

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Abstract

We examine the magnetic field dependence of the muonium(μ^+e^-)-antimuonium(μ^-e^+) conversion in the models which accommodate the dilepton gauge bosons. The effective Hamiltonian for the conversion due to dileptons turns out to be in the $(V - A) \times (V + A)$ form and, in consequence, the conversion probability is rather insensitive to the strength of the magnetic field. The reduction is less than 20% for up to $B \approx 300$ G and 33% even in the large B limit.

PACS number(s): 11.30.Hv, 12.15.Cc, 12.15.Ji, 36.10.Dr

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Muonium, which is a bound state of μ^+ and e^- , can be transformed to antimuonium, a bound state of μ^- and e^+ , if there exists a lepton-number-non-conserving interaction [1]. Feinberg and Weinberg [2] studied the $M - \overline{M}$ conversion with a postulated effective Hamiltonian of $(V - A) \times (V - A)$ form. Later, this process has been studied within the left-right symmetric models and the models with doubly-charged Higgs bosons [3]-[7]. In these models, the effective Hamiltonian for the conversion is expressed either in the $(V - A) \times (V - A)$ form or in the $(V + A) \times (V + A)$ form. Thus far no $M - \overline{M}$ conversion has been observed [8].

Recently, an interesting class of models which have new $SU(2)_L$ -doublet gauge bosons were proposed as extensions of the standard model [9]-[12]. In these models each family of leptons $(l^+, \nu_l, l^-)_L$ transforms as a triplet under the gauge group $SU(3)$ and the total lepton number defined as $L = L_e + L_\mu + L_\tau$ is conserved, while the separate lepton number for each family is not. The new gauge bosons $(X^\mp, X^{\mp\mp})$ carry lepton number $L = \pm 2$. Hence, hereafter, we refer to these gauge bosons as dileptons. The gauge group $SU(3)$ will be, for example, an $SU(3)_l$ in the $SU(15)$ grand unification theory model [10] or an $SU(3)_L$ in the $SU(3)_C \times SU(3)_L \times U(1)_X$ model [12].

The phenomenology on dilepton gauge bosons has been extensively studied. When the doubly-charged dilepton exists, the mixing of muonium and antimuonium is possible through the diagram illustrated in Fig. 1 and thus $M - \overline{M}$ conversion takes place [13]-[15]. In particular, the effective Hamiltonian for the mixing turns out to be in the $(V - A) \times (V + A)$ form. One of the present authors (K.S.) and Fujii and Nakamura calculated the probability for the $M - \overline{M}$ conversion in the models with dileptons and examined the lower mass bound on the doubly-charged dilepton $X^{\pm\pm}$ in Ref.[14]. But the analysis was done in the case of absence of magnetic fields. In this paper we consider the $M - \overline{M}$ conversion in static external magnetic fields and study the field dependence of the conversion probability.

The muonium or antimuonium system in the presence of static external magnetic field \vec{B} is described by the following Hamiltonian,

$$\mathcal{H}_{int} = A \vec{S}_e \cdot \vec{S}_\mu + \mu_B g_e \vec{S}_e \cdot \vec{B} + \mu_B \frac{m_e}{m_\mu} g_\mu \vec{S}_\mu \cdot \vec{B}, \quad (1)$$

where \vec{S}_e , m_e , $g_{e^-} = -g_{e^+}$ and \vec{S}_μ , m_μ , $g_{\mu^+} = -g_{\mu^-}$ are spin, mass, the gyromagnetic ratio of electron (or positron) and μ^+ (or μ^-), respectively, and μ_B is Bohr magneton.

The first term of Eq.(1) is the source of $1S$ hyperfine splitting of the muonium (or antimuonium) system and $A = 1.846 \times 10^{-5} \text{eV}$. Taking the magnetic field direction as the spin-quantization axis, we obtain the muonium energy eigenvalues as follows [16]:

$$\begin{aligned} E_M(1, +1) &= \frac{A}{4} + P \\ E_M(1, -1) &= \frac{A}{4} - P \\ E_M(1, 0) &= -\frac{A}{4}(1 - 2\sqrt{1 + y^2}) \\ E_M(0, 0) &= -\frac{A}{4}(1 + 2\sqrt{1 + y^2}), \end{aligned} \quad (2)$$

with

$$\begin{aligned} P &= \frac{1}{2}\mu_B B(g_{e^-} - g_{\mu^-} \frac{m_e}{m_\mu}) \approx 5.76 \times 10^{-9} B(\text{eV/G}) \\ y &= \frac{1}{A}\mu_B B(g_{e^-} + g_{\mu^-} \frac{m_e}{m_\mu}) \approx 6.30 \times 10^{-4} B(1/\text{G}). \end{aligned} \quad (3)$$

The corresponding eigenstates are expressed in a “natural” basis $|S_\mu^z S_e^z\rangle$ as:

$$\begin{aligned} |1, +1\rangle_M &= |++\rangle_M \\ |1, -1\rangle_M &= |--\rangle_M \\ |1, 0\rangle_M &= c| - + \rangle_M + s| + - \rangle_M \\ |0, 0\rangle_M &= -s| - + \rangle_M + c| + - \rangle_M, \end{aligned} \quad (4)$$

where $|+ - \rangle_M$ means $|S_\mu^z = \frac{1}{2}, S_e^z = -\frac{1}{2}\rangle_M$, etc., and

$$\begin{aligned} c &= \frac{1}{\sqrt{2}}[1 + \frac{y}{\sqrt{1 + y^2}}]^{1/2} \\ s &= \frac{1}{\sqrt{2}}[1 - \frac{y}{\sqrt{1 + y^2}}]^{1/2}. \end{aligned} \quad (5)$$

It is noted that the $(J = 1, J_z = 0)$ state among $1S$ triplet and $1S$ singlet state $(J = 0, J_z = 0)$, which are both energy eigenstates in the absence of external magnetic fields, mix with each other in the presence of \vec{B} and they are not energy eigenstates any more. Thus it is understood that energy eigenstates $|1, 0\rangle$ and $|0, 0\rangle$ are the states which approach to $(J = 1, J_z = 0)$ and $(J = 0, J_z = 0)$ states, respectively, when the magnetic field \vec{B} vanishes. However, $(J = 1, J_z = \pm)$ states among $1S$ triplet remain as energy eigenstates even in the presence of \vec{B} .

Energy eigenvalues and the corresponding eigenstates for the antimuonium system in the presence of external magnetic field \vec{B} are obtained from Eqs.(2)(4) by interchanging $P \leftrightarrow -P$, $y \leftrightarrow -y$ and $c \leftrightarrow s$. Thus the energy eigenvalues for the antimuonium are

$$\begin{aligned} E_{\overline{M}}(1, +1) &= \frac{A}{4} - P \\ E_{\overline{M}}(1, -1) &= \frac{A}{4} + P \\ E_{\overline{M}}(1, 0) &= -\frac{A}{4}(1 - 2\sqrt{1 + y^2}) \\ E_{\overline{M}}(0, 0) &= -\frac{A}{4}(1 + 2\sqrt{1 + y^2}), \end{aligned} \quad (6)$$

and the corresponding eigenstates are

$$\begin{aligned} |1, +1\rangle_{\overline{M}} &= |++\rangle_{\overline{M}} \\ |1, -1\rangle_{\overline{M}} &= |--\rangle_{\overline{M}} \\ |1, 0\rangle_{\overline{M}} &= s| - + \rangle_{\overline{M}} + c| + - \rangle_{\overline{M}} \\ |0, 0\rangle_{\overline{M}} &= -c| - + \rangle_{\overline{M}} + s| + - \rangle_{\overline{M}}. \end{aligned} \quad (7)$$

Now we consider the $M - \overline{M}$ conversion in the presence of static external magnetic fields. First we write down a useful formula for the $M - \overline{M}$ conversion which was derived by Feinberg and Weinberg a long time ago [2]. If there exists an interaction $\mathcal{H}_{M\overline{M}}$ which would yield a matrix element for conversion of M into \overline{M} equal to

$$\langle \overline{M} | \mathcal{H}_{M\overline{M}} | M \rangle = \frac{\Delta}{2}, \quad (8)$$

the mass matrix for the $M - \overline{M}$ system is written as

$$\mathcal{M}_{M\overline{M}} = \begin{pmatrix} E_M & \frac{\Delta}{2} \\ \frac{\Delta}{2} & E_{\overline{M}} \end{pmatrix}. \quad (9)$$

Then the probability for a muonium atom of the state $|M\rangle$ to decay as antimuonium of the state $|\overline{M}\rangle$ at all is given by

$$P(\overline{M}) = \frac{\Delta^2}{2[\lambda^2 + (E_M - E_{\overline{M}})^2 + \Delta^2]}, \quad (10)$$

where $\lambda = G_F^2 m_\mu^5 / 192\pi^3$ is the muon decay rate and G_F is Fermi constant.

Before we study the dilepton contributions to the $M - \overline{M}$ conversion in the presence of static external magnetic fields, we review the case when the effective

Hamiltonian for $M - \overline{M}$ transition is written in the $(V - A) \times (V - A)$ form or $(V + A) \times (V + A)$ form [16][17],

$$\mathcal{H}_{M\overline{M}} = \frac{G_{M\overline{M}}}{\sqrt{2}} [\overline{\mu} \gamma_\lambda (1 \mp \gamma_5) e] [\overline{\mu} \gamma^\lambda (1 \mp \gamma_5) e] + H.c., \quad (11)$$

which arises in the left-right symmetric models and the models with doubly-charged Higgs bosons [3]-[7]. In this case matrix elements for conversion of M into \overline{M} are given in a “natural” basis $|S_\mu^z S_e^z\rangle$ as follows:

$$\begin{aligned} \overline{M} < ++ | \mathcal{H}_{M\overline{M}} | ++ >_M &= \overline{M} < -- | \mathcal{H}_{M\overline{M}} | -- >_M \\ &= \overline{M} < +- | \mathcal{H}_{M\overline{M}} | +- >_M \\ &= \overline{M} < -+ | \mathcal{H}_{M\overline{M}} | -+ >_M \\ &= \frac{\delta}{2} \\ \text{other elements} &= 0, \end{aligned} \quad (12)$$

with

$$\delta = \frac{16G_{M\overline{M}}}{\sqrt{2}\pi a^3}, \quad (13)$$

where a is the Bohr radius of the muonium $(m_r \alpha)^{-1}$ with $m_r^{-1} = m_\mu^{-1} + m_e^{-1}$. Thus we obtain,

$$\begin{aligned} \overline{M} < 1, \pm 1 | \mathcal{H}_{M\overline{M}} | 1, \pm 1 >_M &= \frac{\delta}{2} \\ \overline{M} < 1, 0 | \mathcal{H}_{M\overline{M}} | 1, 0 >_M &= \overline{M} < 0, 0 | \mathcal{H}_{M\overline{M}} | 0, 0 >_M \\ &= \frac{\delta}{2\sqrt{1+y^2}}. \end{aligned} \quad (14)$$

for the matrix elements in the “energy eigenstate” representation. Now it is straightforward from Eqs.(2), (6), (10) and (14) to calculate the probability of a muonium in the $|1, \pm 1\rangle$, $|1, 0\rangle$ and $|0, 0\rangle$ states to decay as antimuonium. The results are [16][17],

$$P^{(1,\pm 1)}(\overline{M}) = \frac{\delta^2}{2[\lambda^2 + 4P^2 + \delta^2]} \quad (15)$$

for the $|1, +1\rangle$ and $|1, -1\rangle$ states and

$$\begin{aligned} P^{(1,0)}(\overline{M}) &= P^{(0,0)}(\overline{M}) \\ &= \frac{\delta^2}{2[(1+y^2)\lambda^2 + \delta^2]} \end{aligned} \quad (16)$$

for the $|1, 0\rangle$ and $|0, 0\rangle$ states.

It is noted that since the $(J = 1, J_z = 0)$ and $(J = 0, J_z = 0)$ states mix with each other in the presence of external magnetic fields, $M - \overline{M}$ conversions from $|1, 0\rangle_M$ to $|0, 0\rangle_{\overline{M}}$ state and from $|0, 0\rangle_M$ to $|1, 0\rangle_{\overline{M}}$ state are also possible. Indeed, from the $M - \overline{M}$ transition matrix elements

$$\begin{aligned}\overline{M}\langle 0, 0|\mathcal{H}_{M\overline{M}}|1, 0\rangle_M &= -\overline{M}\langle 1, 0|\mathcal{H}_{M\overline{M}}|0, 0\rangle_M \\ &= -\frac{y}{\sqrt{1+y^2}}\frac{\delta}{2},\end{aligned}\tag{17}$$

we obtain

$$\begin{aligned}P^{(1,0)\rightarrow(0,0)}(\overline{M}) &= P^{(0,0)\rightarrow(1,0)}(\overline{M}) \\ &= \frac{y^2\delta^2}{2[(1+y^2)\lambda^2 + (1+y^2)^2A^2 + y^2\delta^2]}\end{aligned}\tag{18}$$

for the probability of a muonium of the $|1, 0\rangle_M$ ($|0, 0\rangle_M$) state to decay as antimuonium through the state $|0, 0\rangle_{\overline{M}}$ ($|1, 0\rangle_{\overline{M}}$). However these probabilities are numerically extremely small and can be safely neglected in the following discussion.

The assumption that each state is produced with equal weight at the beginning gives

$$P^{\text{Tot}}(\overline{M}) = \frac{\delta^2}{4[\lambda^2 + 4P^2 + \delta^2]} + \frac{\delta^2}{4[(1+y^2)\lambda^2 + \delta^2]},\tag{19}$$

for the “total” propability of a muonium to decay as antimuonium. The magnetic field dependence of $P^{\text{Tot}}(\overline{M})$ has been studied in Refs. [16][17]. We plot the results for dependence of $P^{\text{Tot}}(\overline{M})$, $\frac{1}{2}P^{(1,1)}(\overline{M})$, and $\frac{1}{2}P^{(1,0)}(\overline{M})$ on B in Fig.2. Note that the probabilities are normalized by $P^{\text{Tot}}(\overline{M})|_{B=0}$ and $G_{M\overline{M}}$ is taken to be $0.1G_F$.

In the presence of static external magnetic fields, the degeneracy between the $|1, +1\rangle_M$ and $|1, +1\rangle_{\overline{M}}$ states (the $|1, -1\rangle_M$ and $|1, -1\rangle_{\overline{M}}$ states) breaks down and the generated energy difference severely suppresses the conversion. In fact, the probability $P^{(1,\pm 1)}(\overline{M})$ becomes negligibly small when B is in the order of 10^{-1} G (see Fig.2-b). On the other hand, the $|1, 0\rangle_M$ and $|1, 0\rangle_{\overline{M}}$ states (the $|0, 0\rangle_M$ and $|0, 0\rangle_{\overline{M}}$ states) remain degenerate and thus the conversion persists up to the fields in the order of 10^3 G. In the limit of large B , the $|1, 0\rangle_M$ state becomes a pure $|-+\rangle_M$ while the $|1, 0\rangle_{\overline{M}}$ state becomes a pure $|+-\rangle_{\overline{M}}$, and thus the matrix element $\overline{M}\langle 1, 0|\mathcal{H}_{M\overline{M}}|1, 0\rangle_M$ vanishes. Hence the probability $P^{(1,0)}(\overline{M})$ reduces to zero in this limit (see Fig.2-c below). By the same reason, $P^{(0,0)}(\overline{M})$

vanishes in the large B limit. Finally we see from Fig.2-a that in the case of the effective Hamiltonian being in the $(V - A) \times (V - A)$ form or $(V + A) \times (V + A)$ form and $G_{M\overline{M}} = 0.1G_F$, the $M - \overline{M}$ conversion probability is reduced to 50% at a field strength as low as 0.26 G, to 35.8% at $B = 1$ kG and to 1.2% at $B = 1$ T. The dependence of the normalized probabilities on the coupling strength $G_{M\overline{M}}$ is negligibly small for $G_{M\overline{M}} < 1G_F$.

Next we consider the $M - \overline{M}$ conversion in models with dileptons. The gauge interaction of dileptons with leptons is given by [18]

$$\begin{aligned} \mathcal{L}_{int} = & -\frac{g_{3l}}{2\sqrt{2}}X_\mu^{++}l^TC\gamma^\mu\gamma_5l - \frac{g_{3l}}{2\sqrt{2}}X_\mu^{--}\bar{l}\gamma^\mu\gamma_5C\bar{l}^T \\ & + \frac{g_{3l}}{2\sqrt{2}}X_\mu^{+}l^TC\gamma^\mu(1-\gamma_5)\nu_l + \frac{g_{3l}}{2\sqrt{2}}X_\mu^{-}\bar{\nu}_l\gamma^\mu(1-\gamma_5)C\bar{l}^T, \end{aligned} \quad (20)$$

where $l = e, \mu, \tau$, and C is the charge-conjugation matrix. The gauge coupling constant g_{3l} is given approximately by $g_{3l} = 1.19e$ for the SU(15) GUT model [10] and by $g_{3l} = g_2 = 2.07e$ for the $SU(3)_L \times U(1)_X$ model [12], where e and g_2 are the electric charge and the $SU(2)_L$ gauge coupling constant, respectively. It is noted that the vector currents which couple to doubly-charged dileptons $X^{\pm\pm}$ vanish due to Fermi statistics. Through the doubly-charged-dilepton-exchange diagram illustrated in Fig. 1, we obtain the following effective Hamiltonian for the $M - \overline{M}$ conversion,

$$\mathcal{H}_{M\overline{M}}^{Di} = \frac{G_{M\overline{M}}^{Di}}{\sqrt{2}}[\bar{\mu}\gamma_\lambda(1-\gamma_5)e][\bar{\mu}\gamma^\lambda(1+\gamma_5)e] + H.c. \quad (21)$$

where $G_{M\overline{M}}^{Di}/\sqrt{2} = -g_{3l}^2/(8M_{X^{\pm\pm}}^2)$ and $M_{X^{\pm\pm}}$ is the doubly-charged dilepton mass. This form is obtained from Eq.(20) and with help of the Fierz transformation. It should be noted that the above effective Hamiltonian is in the $(V - A) \times (V + A)$ form. The most stringent lower mass bound for the doubly-charged dileptons at present is $(M_{X^{\pm\pm}}/g_{3l}) > 340$ GeV (95%C.L.) [18]. This gives $G_{M\overline{M}}^{Di} < 0.13G_F$.

With this effective Hamiltonian, we find that the matrix elements for conversion of M into \overline{M} are given in a “natural” basis $|S_\mu^z S_e^z\rangle$ as follows:

$$\begin{aligned} \overline{M} < ++ | \mathcal{H}_{M\overline{M}}^{Di} | ++ >_M &= \overline{M} < -- | \mathcal{H}_{M\overline{M}}^{Di} | -- >_M = \frac{\hat{\delta}}{2} \\ \overline{M} < +- | \mathcal{H}_{M\overline{M}}^{Di} | +- >_M &= \overline{M} < -+ | \mathcal{H}_{M\overline{M}}^{Di} | -+ >_M = -\frac{\hat{\delta}}{2} \\ \overline{M} < +- | \mathcal{H}_{M\overline{M}}^{Di} | -+ >_M &= \overline{M} < -+ | \mathcal{H}_{M\overline{M}}^{Di} | +- >_M = \hat{\delta} \end{aligned}$$

$$\text{other elements} = 0, \quad (22)$$

where

$$\hat{\delta} = -\frac{8G_{MM}^{Di}}{\sqrt{2}\pi a^3}. \quad (23)$$

Since \mathcal{H}_{MM}^{Di} is in the $(V-A) \times (V+A)$ form, the matrix elements $\overline{M} < ++ |\mathcal{H}_{MM}^{Di}| + + >_M$ and $\overline{M} < +- |\mathcal{H}_{MM}^{Di}| + - >_M$ take different values, and $\overline{M} < +- |\mathcal{H}_{MM}^{Di}| - + >_M$ and $\overline{M} < - + |\mathcal{H}_{MM}^{Di}| + - >_M$ do not vanish.

In terms of the “energy eigenstates”, the matrix elements for $M - \overline{M}$ conversion are written as ,

$$\begin{aligned} \overline{M} < 1, \pm 1 |\mathcal{H}_{MM}^{Di}| 1, \pm 1 >_M &= \frac{\hat{\delta}}{2} \\ \overline{M} < 1, 0 |\mathcal{H}_{MM}^{Di}| 1, 0 >_M &= (1 - \frac{1}{2\sqrt{1+y^2}})\hat{\delta} \\ \overline{M} < 0, 0 |\mathcal{H}_{MM}^{Di}| 0, 0 >_M &= -(1 + \frac{1}{2\sqrt{1+y^2}})\hat{\delta}. \end{aligned} \quad (24)$$

It is interesting to note that neither $\overline{M} < 1, 0 |\mathcal{H}_{MM}^{Di}| 1, 0 >_M$ nor $\overline{M} < 0, 0 |\mathcal{H}_{MM}^{Di}| 0, 0 >_M$ vanishes in the large B (i.e., large y) limit.

Again using the formula (10), we obtain the following probabilities of a muonium to decay as antimuonium in the models with dileptons:

$$P_{Di}^{(1,\pm 1)}(\overline{M}) = \frac{\hat{\delta}^2}{2[\lambda^2 + 4P^2 + \hat{\delta}^2]} \quad (25)$$

for the $|1, \pm 1 >_M$ states,

$$P_{Di}^{(1,0)}(\overline{M}) = \frac{(2 - \frac{1}{\sqrt{1+y^2}})^2 \hat{\delta}^2}{2[\lambda^2 + (2 - \frac{1}{\sqrt{1+y^2}})^2 \hat{\delta}^2]} \quad (26)$$

for the $|1, 0 >_M$ state and finally

$$P_{Di}^{(0,0)}(\overline{M}) = \frac{(2 + \frac{1}{\sqrt{1+y^2}})^2 \hat{\delta}^2}{2[\lambda^2 + (2 + \frac{1}{\sqrt{1+y^2}})^2 \hat{\delta}^2]} \quad (27)$$

for the $|0, 0 >_M$ state.

As before we assume that each state is produced with equal weight at the beginning, and we obtain,

$$P_{Di}^{\text{Tot}}(\overline{M}) = \frac{\hat{\delta}^2}{4[\lambda^2 + 4P^2 + \hat{\delta}^2]} + \frac{(2 - \frac{1}{\sqrt{1+y^2}})^2 \hat{\delta}^2}{8[\lambda^2 + (2 - \frac{1}{\sqrt{1+y^2}})^2 \hat{\delta}^2]} + \frac{(2 + \frac{1}{\sqrt{1+y^2}})^2 \hat{\delta}^2}{8[\lambda^2 + (2 + \frac{1}{\sqrt{1+y^2}})^2 \hat{\delta}^2]}. \quad (28)$$

for the “total” probability of a muonium to decay as antimuonium. In the limit of $B = 0$, we have

$$\begin{aligned} P_{Di}^{\text{Tot}}(\overline{M})|_{B=0} &= \frac{3\hat{\delta}^2}{8[\lambda^2 + \hat{\delta}^2]} + \frac{9\hat{\delta}^2}{8[\lambda^2 + 9\hat{\delta}^2]} \\ &\approx \frac{3\hat{\delta}^2}{2\lambda^2}, \end{aligned} \quad (29)$$

which is the result first obtained in Ref. [14].

In Fig.3 we plot the magnetic field dependence of $P_{Di}^{\text{Tot}}(\overline{M})$, $\frac{1}{2}P_{Di}^{(1,1)}(\overline{M})$, $\frac{1}{4}P_{Di}^{(1,0)}(\overline{M})$, and $\frac{1}{4}P_{Di}^{(0,0)}(\overline{M})$. They are all normalized by $P_{Di}^{\text{Tot}}(\overline{M})|_{B=0}$ and we take $G_{M\overline{M}}^{Di} = 0.1G_F$. As in the case of $P^{(1,\pm 1)}(\overline{M})$, the probability $P_{Di}^{(1,\pm 1)}(\overline{M})$ becomes negligibly small when B reaches the order of 10^{-1}G since the magnetic field breaks the degeneracy of the $|1, +1\rangle_M$ and $|1, +1\rangle_{\overline{M}}$ states (see Fig.3-b). However, the B -dependences of $P_{Di}^{(1,0)}(\overline{M})$ and $P_{Di}^{(0,0)}(\overline{M})$ are quite different from those of $P^{(1,0)}(\overline{M})$ and $P^{(0,0)}(\overline{M})$ (see Fig.3-c,d). Firstly, the $M - \overline{M}$ conversion through the channel $|0, 0\rangle_M \rightarrow |0, 0\rangle_{\overline{M}}$ is much preferred. Thus $P_{Di}^{(0,0)}(\overline{M})$ gives a dominant contribution to $P_{Di}^{\text{Tot}}(\overline{M})$. Secondly, $P_{Di}^{(1,0)}(\overline{M})$ and $P_{Di}^{(0,0)}(\overline{M})$ remain finite in the large B limit. This is due to the fact that the matrix elements $\overline{M} < 1, 0 | \mathcal{H}_{M\overline{M}}^{Di} | 1, 0 \rangle_M$ and $\overline{M} < 0, 0 | \mathcal{H}_{M\overline{M}}^{Di} | 0, 0 \rangle_M$ do not vanish in the large B limit when the effective Hamiltonian is in the $(V - A) \times (V + A)$ form. Interestingly enough, $P_{Di}^{(1,0)}(\overline{M})$ starts to increase around $B = 1 \text{ kG}$ and partially compensates the decrease of $P_{Di}^{(0,0)}(\overline{M})$ in the region $B > 1 \text{ kG}$. Summing up each contributions, we find that $P_{Di}^{\text{Tot}}(\overline{M})$ is rather insensitive to the static external magnetic field. In fact Fig.3-a shows that $P_{Di}^{\text{Tot}}(\overline{M})$ is lowered to 83% in the region $0.2 \text{ G} < B < 300 \text{ G}$ and only to 67% in the large B limit. At $B = 1 \text{ kG}$ (1 T) the reduction is 22.4% (32.9%). Again the dependence of the normalized probabilities on the coupling strength $G_{M\overline{M}}^{Di}$ is negligibly small for $G_{M\overline{M}}^{Di} < 1G_F$.

In conclusion, we have studied the magnetic field dependence of the $M - \overline{M}$ conversion in the models with dileptons. We have found that the conversion is rather

insensitive to the strength of the magnetic fields. If an experiment is performed in a magnetic field of 1 T and if a bound for the conversion probability $P(\overline{M}) < 10^{-10}$ is gained [17], then a bound for the coupling strength, $G_{M\overline{M}} < 1.8 \times 10^{-2} G_F$, is obtained for the usual $(V \mp A) \times (V \mp A)$ type-Hamiltonian. On the other hand, the models with dileptons give a more stringent bound $G_{M\overline{M}}^{Di} < 2.8 \times 10^{-3} G_F$.

Acknowledgements

K.S. would like to thank Professor G. zu Putlitz for the hospitality extended to him when he visited Physikalisches Institut der Universität Heidelberg in the summer of 1994 and for useful discussions. We would like to thank Professor K. Jungmann for introducing the work of Refs. [16][17] to us, which inspired us to start this work.

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Figure caption

Figure 1

The doubly-charged dilepton exchange diagram for muon-antimuonium conversion. The arrows show the flow of lepton number.

Figure 2

The magnetic field dependence of the $M - \overline{M}$ conversion probability with an effective $(V \mp A) \times (V \mp A)$ type-Hamiltonian: (a) $P^{\text{Tot}}(\overline{M})$; (b) $\frac{1}{2}P^{(1,1)}(\overline{M})$; (c) $\frac{1}{2}P^{(1,0)}(\overline{M})$. They are all normalized by $P^{\text{Tot}}(\overline{M})|_{B=0}$ and $G_{M\overline{M}} = 0.1G_F$ is assumed.

Figure 3

The magnetic field dependence of the $M - \overline{M}$ conversion probability in models with dileptons: (a) $P_{Di}^{\text{Tot}}(\overline{M})$; (b) $\frac{1}{2}P_{Di}^{(1,1)}(\overline{M})$; (c) $\frac{1}{4}P_{Di}^{(1,0)}(\overline{M})$; (d) $\frac{1}{4}P_{Di}^{(0,0)}(\overline{M})$. They are all normalized by $P_{Di}^{\text{Tot}}(\overline{M})|_{B=0}$ and $G_{M\overline{M}}^{Di} = 0.1G_F$ is assumed.

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